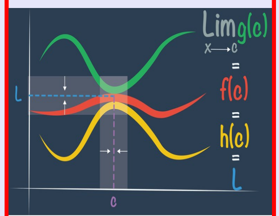
Calculus I

Lecture 50



Feb 19-8:47 AM

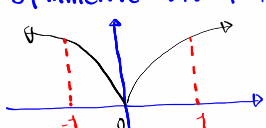
The average value for a cont. function $f(x)$ on $[a,b]$ is given by

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

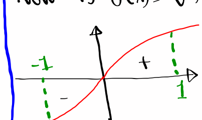
$\boxed{\text{Find } f_{ave} \text{ for } f(x) = \sqrt[3]{x^2} \text{ over } [-1,1].}$

$f(-x) = \sqrt[3]{(-x)^2} = \sqrt[3]{x^2} = f(x)$

$f(x)$ is an even function
Symmetric w/t Y-axis.



Now if $f(x) = \sqrt[3]{x}$



$f_{ave} = 0$ on $[-1,1]$ verify that on your own.

$$f_{ave} = \frac{1}{1-(-1)}\int_{-1}^{1}\sqrt[3]{x^2}\,dx$$

$$= \frac{1}{2}\int_{-1}^{1} x^{2/3}\,dx \qquad \frac{2}{3}+1=\frac{5}{3}$$

$$= \frac{1}{2}\cdot\frac{x^{5/3}}{5/3}\Big|_{-1}^{1}$$

$$= \frac{1}{2}\cdot\frac{3}{5}\,x\sqrt[3]{x^2}\Big|_{-1}^{1}$$

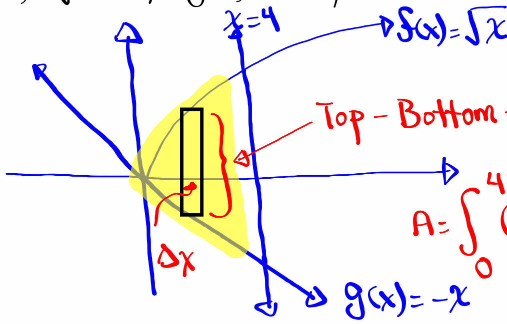
$$= \frac{3}{10}\left[1\sqrt[3]{1^2} - (-1)\sqrt[3]{(-1)^2}\right]$$

$$= \frac{3}{10}\left[1+1\right] = \frac{6}{10} = \boxed{\frac{3}{5}}$$

May 14-9:48 AM

Find the area of the region enclosed by

$f(x) = \sqrt{x}$, $g(x) = -x$, and $x = 4$. Drawing Required.



Top − Bottom $= \sqrt{x} - (-x) = \sqrt{x} + x$

$\sqrt{x} \to x^{1/2}$

$$A = \int_0^4 (\sqrt{x} + x)\,dx$$

$$= \left[\frac{x^{3/2}}{3/2} + \frac{x^2}{2}\right]\Bigg|_0^4 = \left(\frac{2}{3}\cdot x\sqrt{x} + \frac{1}{2}x^2\right)\Bigg|_0^4$$

$$= \frac{2}{3}\cdot 4\sqrt{4} + \frac{1}{2}(4)^2 - 0$$

$$= \frac{16}{3} + 8 = \frac{16+24}{3} = \boxed{\frac{40}{3}}$$

May 15-8:54 AM

Find the area enclosed by $x = y^4$, $y = \sqrt{2-x}$,

and $y = 0$. Drawing Required.

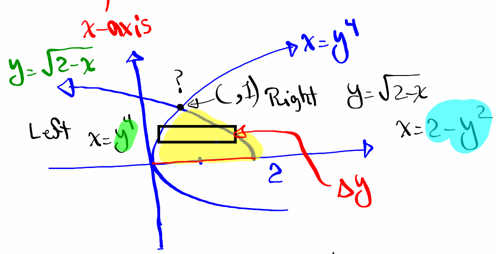
$x \geq 0$; $y \geq 0$

$y = 0 \to$ x-axis

$y^2 = 2 - x$

$x = 2 - y^2$

Sideway Parabola open left, shift 2 units right



$$A = \int (\text{Right} - \text{left})\,dy = \int_0^1 (2 - y^2 - y^4)\,dy$$

$2 - y^2 = y^4$

$y^4 + y^2 - 2 = 0$

$(y^2+2)(y^2-1) = 0$

$y^2+2 \neq 0 \quad y^2-1=0$

$y = \pm 1$

$$= \left(2y - \frac{y^3}{3} - \frac{y^5}{5}\right)\Bigg|_0^1$$

$$= 2 - \frac{1}{3} - \frac{1}{5} = \frac{30-5-3}{15} = \boxed{\frac{22}{15}}$$

May 15-9:02 AM

Use chain Rule to find

$$\frac{d}{dx}\left[\cos(x^3-1)\right] = -\sin(x^3-1)\cdot 3x^2$$

So $$\int -3x^2\cdot\sin(x^3-1)\,dx = \cos(x^3-1)+C$$

Use chain rule to find

$$\frac{d}{dx}\left[(x^2-4x+3)^5\right] = 5(x^2-4x+3)^4\cdot(2x-4)$$

So $$\int 5(2x-4)\cdot(x^2-4x+3)^4\,dx = (x^2-4x+3)^5+C$$

May 15-9:15 AM

Use chain rule to find

$$\frac{d}{dx}\left[F(g(x))\right] = F'(g(x))\cdot g'(x)$$

So $$\int F'(g(x))\cdot g'(x)\,dx = F(g(x))+C$$

If $u=g(x)$ is a diff. function and $f(x)$ is Cont. on Interval I, then

$$\int f(g(x))\cdot g'(x)\,dx = \int f(u)\,du$$

Subs. Method

May 15-9:21 AM

$$\int 2x\cos(x^2)\,dx = \int \cos u\, du$$

$u = x^2$

$du = 2x\,dx$

$$= \sin u + C$$

$$= \boxed{\sin(x^2) + C}$$

to verify

$$\frac{d}{dx}\left[\sin x^2 + C\right] = \cos x^2 \cdot 2x + 0 = 2x\cos x^2$$

May 15-9:26 AM

$$\int x^3(\underbrace{1+x^4}_{})^5\,dx = \int u^5\,\frac{du}{4} = \frac{1}{4}\int u^5\,du$$

$u = 1 + x^4$

$du = 4x^3\,dx$

$\frac{du}{4} = x^3\,dx$

$$= \frac{1}{4}\cdot\frac{u^6}{6} + C$$

$$= \boxed{\frac{1}{24}(1+x^4)^6 + C}$$

Let's verify

$$\frac{d}{dx}\left[\frac{1}{24}(1+x^4)^6 + C\right] = \frac{1}{\cancel{24}}\cdot\cancel{6}(1+x^4)^5\cdot\cancel{4}x^3 + 0$$

$$= \boxed{x^3(1+x^4)^5}$$

May 15-9:30 AM

find $\int (2x-1)^9 \, \underline{dx}$

Let $u = 2x-1$

$du = 2\,dx$

$\frac{du}{2} = dx$

$= \int u^9 \frac{du}{2}$

$= \frac{1}{2}\int u^9 du = \frac{1}{2}\cdot\frac{u^{10}}{10} + C = \frac{1}{20}u^{10} + C$

$= \boxed{\frac{1}{20}(2x-1)^{10} + C}$

To verify

$\frac{d}{dx}\left[\frac{1}{20}(2x-1)^{10} + C\right] = \frac{1}{\cancel{20}}\cdot 10(2x-1)^9 \cdot 2 + 0 = (2x-1)^9$

May 15-9:36 AM

find $\int \underbrace{\sin^4 x}_{u^4} \underbrace{\cos x \, dx}_{du}$

Let $u = \sin x$

$du = \cos x \, dx$

$= \int u^4 du = \frac{u^5}{5} + C$

$= \frac{1}{5}\sin^5 x + C$

To verify $\frac{d}{dx}\left[\frac{1}{5}\sin^5 x + C\right] = \cdots = \sin^4 x \cos x$

May 15-9:42 AM

Find f_{ave} for $f(x) = \cos^4 x \sin x$ over $[0, \pi]$.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{\pi - 0}\int_0^{\pi} \cos^4 x \sin x\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \cos^4 x \sin x\,dx$$

$u = \cos x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$x = 0 \quad u = \cos 0 = 1$

$x = \pi \quad u = \cos \pi = -1$

$$= \frac{1}{\pi}\int_1^{-1} u^4 \cdot (-du)$$

$$= \frac{-1}{\pi} \cdot \frac{u^5}{5}\Big|_1^{-1}$$

$$= \frac{-1}{5\pi}\left[(-1)^5 - (1)^5\right] = \frac{-1}{5\pi}\left[-1 - 1\right] = \frac{-1}{5\pi}(-2) = \boxed{\frac{2}{5\pi}}$$

May 15-9:46 AM